Dr. Usha Kumari
physics Dept.

# Sound

## Unit :- IV



## "Differential Equation of a Wave Motion".

Equ$^n$ of plane progressive wave motion

$$y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

particle velocity $U = \frac{dy}{dx}$

$$\Rightarrow \frac{dy}{dt} = \frac{2\pi v}{\lambda} a \cos \frac{2\pi}{\lambda}(vt - x) \qquad \text{(I)}$$

Relation bet$^n$ particle velocity & wave velocity :-

$$\frac{dy}{dx} = -\frac{2\pi}{\lambda} a \cos \frac{2\pi}{\lambda}(vt - x) \qquad \text{(ii)}$$

$$\therefore \quad U = \frac{dy}{dt} = -v\frac{dy}{dx} \qquad \text{(iii)}$$

Acceleration of particle,

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2} a \sin \frac{2\pi}{\lambda}(vt - x)$$

$$= -\frac{4\pi^2 v^2}{\lambda} y \qquad \text{(IV)}$$

& Similarly, differentiating equ$^n$(ii) under the same article, for strain or compression, again with respect to $x$, we have :-

Rate of change of Compensation with distance,

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2}{\lambda^2} a \sin \frac{2\pi}{\lambda}(vt - x)$$

$$= -\frac{4\pi^2}{\lambda^2} y \qquad \text{——— (V)}$$

from equn (IV) & (V) we have

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \qquad \text{——— (Vi)}$$

This is called differential equn of plane.

Thus we can say that:-
particle acceleration at a point
= (Wave velocity)$^2$ X (Curvature of displacement curve at that point)

* For one dimensional wave :- Differential Equation is

$$\frac{d^2\psi}{dt^2} = v^2 \frac{d^2\psi}{dx^2}$$

* For Three dimensional wave differential equn is

$$\frac{d^2\psi}{dt^2} = v^2 \nabla^2 \psi$$

Where $\nabla^2 \psi = \frac{\delta^2 \psi}{\delta x^2} + \frac{\delta^2 \psi}{\delta y^2} + \frac{d^2 \psi}{\delta z^2}$.

One possible solution of equation is

$$\psi = a \sin(\omega t - kr + \theta)$$

Where $\psi$ is the displacement at a point $r$ at an instant $t$ and $k$, the propagation vector whose magnitude is equal to $k = 2\pi/\lambda$ (the propagation constant) and direction the same as that of the wave propagation.

—x—